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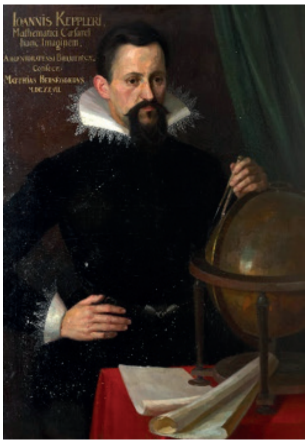
Johannes Kepler and his three laws of planetary motion

Even if Kepler's date of birth, December 27, 1571, is given according to the Julian calendar, one can celebrate its 450th anniversary in 2021 since the Gregorian calendar was only introduced in 1582.

After his studies in theology, astronomy, and mathematics at the University at Tübingen, Kepler received a teaching position at the Protestant "Stiftsschule" at Graz in 1594. In reaction to his book *Mysterium cosmographicum* of 1597 Tycho Brahe (1546–1601) offered him a position at Prague to analyze Brahe's astronomical observation data. Kepler followed this offer in 1600 and even became Brahe's successor as Imperial court mathematician to Emperor Rudolf II (1552–1612). The latter hired him for setting up the *Tabulae Rudolphinae*, a collection of tables of positions of celestial objects and rules which made it possible to calculate such positions in advance. [1]



Johannes Kepler, at the latest in 1620

As a by-product of this Kepler found his laws of planetary motion:

1. **The orbit of a planet is an ellipse with Sun at one of the two foci.**
   (Kepler stated this in his *Astronomia nova* of 1609 only for Mars and without the information on the focus and in his *Epitome Astronomiae Copernicanae* of 1622 in full for all planets of the Solar system.)
2. **The line segment joining a planet and Sun sweeps out equal areas during equal intervals of time.**
   (This is stated in *Astronomia nova* only for Mars and in *Epitome Astronomiae Copernicanae* in general.)
3. **The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.**
   (This is given in Kepler's *Harmonices Mundi* of 1619.)

Even if these laws were found for objects close to Earth, they can be used to detect extra solar planets and measure masses, in particular "dark masses" which cannot be detected elsewise.

After the death of Rudolf II in 1612 Kepler left Prague to become "Landschaftsmathematiker" for Upper Austria at Linz. During the years he got problems with both Christian churches so that he had to escape the town in 1626. But he was able to rescue the manuscript of the *Tabulae Rudolphinae* which were printed in Ulm next year.

That year he also became astrologer to Albrecht von Wallenstein (1583–1634). Kepler died on November 15, 1630 at Regensburg where he wanted to use the "Reichstag" meeting in order to collect outstanding debts.

The general background

At that time the question whether the system of the planets and Sun was geocentric – all celestial objects rotate around Earth – or heliocentric – all around Sun – had been fueled by Nicolaus Copernicus' (1473–1543) book *De revolutionibus orbium coelestium*. So, astronomy had become interesting also for princes. Rudolf II, for example, Emperor of the Holy Roman Empire of German Nation, called Brahe to his capital Prague. Brahe, as already mentioned, in turn called Kepler, not in order to find ellipses but to back up Brahe's own idea for the planetary system which had no ellipses and was not even a true Copernican system.

Even if the Imperial family was not too happy with the interests of Rudolf II in the arts and sciences, astronomy could also be a way to get recognition as a prince, like for Landgrave Wilhelm IV of Hessen-Kassel (1532–1592), who got things going decades before Kepler: Around 1560 he himself started astronomical observations and founded the first fixed observatory in state hands in Europe in modern times. Following Brahe's visit at Kassel in 1575 Wilhelm IV wrote a letter of recommendation to King Frederick II (1534–1588) of Denmark and Norway. In the wake of this, Brahe received the island of Hven in the Öresund as fiefdom and the financial means for his astronomical observatories, which amounted between 1 and 2 percent of the king's revenues.

These activities were not only successful as a scientific but also as a political enterprise: A lot of the influence of the house of Hessen had been gambled away by the father of Wilhelm IV. The scientific activities, however, raised his prestige among the princes, and the astronomical clocks and globes produced at Kassel interested Emperor Rudolf II.

1 Note that another Kepler portrait has recently been questioned, see Physics Today **74**(9), 10 (2021). Also Kepler's bust in the Bavarian Walhalla shows a (further) wrong person.

Jost Bürgi

In this respect Bürgi was of utmost importance: He had been born on February 28, 1552 at Lichtensteig, now: Canton St. Gallen, Switzerland, and was trained as watchmaker, but had neither command of Greek nor Latin. When he started to design scientific instruments and astronomical clocks at Kassel on July 25, 1579, his craftsmanship was already excellent: He improved the instruments so that the accuracy of the measurements became comparable to those at Brahe's observatories [2].

Jost Bürgi in 1619

In order to improve his skills, Bürgi read scientific texts written in German and exchanged his knowledge with other astronomers and mathematicians, often in a kind of barter. For example, he helped Nicolaus Reimers Ursus (1551–1600) with mathematics and in turn the latter made the first translation of Copernicus' *De revolutionibus* into German for him. Even with Kepler Bürgi made such a bargain.

By this Bürgi was no longer forced to treat positions of celestial objects as static but could also calculate and then simulate their motion. Before him, celestial globes had mainly been storages for astronomical data. By putting clockworks into such a globe and connecting them with pointers on its surface, however, Bürgi built machines which dynamically showed the position of celestial bodies: The storage had become an analog computer, for example, the "Himmelglobus" of 1594 kept in the Schweizerische Landesmuseum at Zürich.

Simply reading off positions from these simulation devices made them attractive for persons that disliked to bother with calculations. Emperor Rudolf II got interested in them and Bürgi had several audiences with him at Prague. In turn, when Wilhelm IV died, his successor renewed the contract with Bürgi. In 1604, however he moved to Prague for the same position that he had held at Kassel and that he hold under Rudolf II and his two successors.

So Bürgi and Kepler worked together at the Imperial court from 1604 to 1612 when the latter left for Linz. This does not mean, however, that Bürgi was subordinate to Kepler as is sometimes claimed: He had a better salary than Kepler, and on February 3, 1611 he was ennobled.

In the end of 1631 Bürgi traveled back to Kassel where he died in January 1632. His funeral took place there on January 31, 1632.

Tycho Brahe

Even higher was the worldly position into which Tyge, latinized as Tycho, Brahe was born at Knutstrup Castle in Schonen on December 14, 1546. His noble family was highly influential in Denmark to which Schonen belonged at that time, and he took up university studies which should prepare him for a high political function. But, on August 21, 1560 he observed a partial Solar eclipse, which lead him to devote his life to astronomy. On November 11, 1572, he was the first one to identify definitely a Nova in Europe.

As consequence of the letter of recommendation by Wilhelm IV King Frederick II gave Brahe both the island of Hven and financial means to build the observatories Uraniborg and Stjerneborg there. Because of the size of the instruments and their precision the measurements were about a factor 5 to 10 more exact than those of their predecessors and came to an accuracy of about 1 angular minute. Since they were mostly made of wood, however, the humidity from the sea caused problems. And: Brahe had no telescopes.

The death of Frederick II in 1588 brought a shortening of the financial means. As a consequence, Brahe left Hven together with the logs of his observations and finally followed the offer of Rudolf II to come to Prague in June 1599. As already mentioned, in 1600 Kepler became his assistant, but Brahe died on October 24, 1601.

Brahe had developed his own model of the planetary system, a hybrid of the geocentric and the heliocentric one: Only Sun and Moon circled around Earth while the remaining planets moved on circular orbits around Sun. This system gave rise to the hope that it could work without the epicycles that the other two systems had to use: In order to bring measured and calculated positions of the celestial bodies into agreement, simple circular paths around Earth had been substituted by paths where a second circular path, the "epicycle", on which the planet ran, was centered on the edge of the first circle.

It was clear for Brahe that there were no authorities from Antiquity to support his system. Therefore, he wanted to prove its validity by empirical evidence from his measured data.

How Brahe's data became Kepler's laws

Therefore, when Kepler came to Prague, Brahe assigned him the task to calculate the orbit of a planet. By lucky chance – because of its rather large eccentricity –, Mars was waiting to be done. Kepler took up this task under the assumption of circular paths of Earth and Mars around Sun, which are uniformly run through, but with the center not necessarily in the Sun. Furthermore, Kepler wished that there should be no epicycles.

Kepler thought at first that this task would take only a few days. It lasted, however, five years to find the best approximation – and then he had to realize that there still was a difference of eight angular minutes between measured and calculated positions: "Who would think that this could happen?" he asks in the beginning of Caput XIX of his *Astronomia Nova*. He did not give up, however, but took seriously Brahe's idea of proof by data. Furthermore, he did still not want to use epicycles.

So, he had to allow another curve for the orbits: Ellipses had been known since Antiquity. But Plato (428/427–348/347 B.C.) had stated that the objects that the creator of the world had made could only move uniformly on a circular path. Therefore, each astronomer before Kepler had shrunk back from other kinds of motions. (Because of his missing classical education, Bürgi, however, was not afraid of using elliptic

2 Cf. Andreas Schrimpf, Frank Verbunt: arXiv:2103.03034 and arXiv:2103.10801

gears in his astronomical clocks and globes.)

But Kepler felt more or less forced to use ellipses, as explicitly stated in his first law of planetary motion and implicitly in his second law. He expressed the first ideas of this “area law” already in a letter to David Fabricius (1564–1617) of July 4, 1603 and it helped him in the following to test his first law.

Kepler’s third law was only stated in his *Harmonices Mundi* of 1619 – just in the same chapter as his argument that there could only be six planets because five Platonic solids have to be put between them.

On the relation between Kepler and Bürgi

Kepler’s remark on Bürgi from the foreword of *Tabulae Rudolphinae* that the latter did not rear up his logarithms, but forsook them, is sometimes interpreted in the way that Bürgi was an arithmetic servant to Kepler who did not fulfil his duties. Recent findings and publications, however, have swung the pendulum to the opposite and sometimes lead to the impression that Bürgi had all what was necessary for Kepler’s laws besides a command of Latin.

Taking into account the new view of simulation as a third way besides experiment and theory, one might paint the situation in a more differentiated way:

Kepler was the natural philosopher, who wanted to ***understand*** the celestial system. Bürgi was the engineer, who wanted to design machines that ***reproduce*** and ***predict*** its behavior.

References

Kepler’s collected works are available at
https://kepler.badw.de/das-projekt.html.